An Adaptive Multiresolution Approach to the Simulation of Planar Structures

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***Abstract*—An efficient approach for the full-wave analysis of printed structures is presented. It is based on the use of vector multiresolution (MR) functions in conjunction with the impedance matrix compression (IMC) technique, which leads to a reduced set of iteratively selected basis functions. The multilevel structure of the functions makes the matrix compression possible and also allows its further sparsification, with the subsequent reduction of the computational time and the matrix memory occupancy. Numerical results confirm the efficiency of the technique.**

***Index Terms*—Adaptive schemes, multiresolution, planar structures, printed circuits.**

I. Introduction

A new approach has been recently introduced [1] for the method of moments (MoM) analysis of printed structures. It is based on the generation of multiresolution (MR) vector functions that result in a fast convergence of iterative solvers, but especially in the possibility of drastically sparsifying the MoM matrix.

The MR functions are intrinsically multilevel and hierarchical in each region of the structure, i.e., one achieves a finer resolution by *adding* functions belonging to higher levels. This makes the use of MR ideally suited to adaptive iterative schemes, i.e., which "select" the "minimum" set of basis functions necessary to achieve a given accuracy. This is contrasted to standard (e.g., rooftop or RWG) bases, in which a refinement step requires a different mesh and different functions over the entire region involved in the refinement, with minimal re-use of the MoM matrix computed at the previous step.

In this work, we combine the vector MR functions introduced in [1] and the adaptive iterative basis functions selection introduced in [2], [3], termed "impedance matrix compression" (IMC). The resulting scheme is applied to the analysis of printed structures.

The IMC was devised for the solution of scattering problems and employs the properties of the (scalar) wavelet functions to iteratively select only the basis functions needed to describe the solution to within a certain accuracy. The class of problems of interest here have some important differences from those considered in [2], [3], notably the vector nature of the current and the presence of subwavelength geometrical details. This makes the application of the original IMC procedure not straightforward: the main issues and changes that have to be introduced are reported in Section III. The numerical results reported in the Section IV will demonstrate the efficiency of the obtained procedure. Preliminary results were presented in [4].

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II. Background

We begin by briefly summarizing the IMC procedure in [2], [3]. The selection of the "minimum" set of basis functions follows an iterative refinement process: at each step n, one determines a set of basis functions that has a better resolution than that at the $(n-1)$-th step; numerical efficiency requires that the set at step n includes the set of step $(n-1)$. In this way, the MoM matrix at step n is obtained from that at step $(n-1)$ by computing only the term related to the newly introduced terms.

The iterative procedure starts with an initial set of basis functions, which is assumed to be a "significant enough" set for describing the current. Solving for the coefficients of only these functions, one gets a crude approximation of the solution current; using it, an "error signal" is computed, which describes the error in complying with the boundary-condition requirements of the EM problem. This error signal is then analyzed and gives indications as to which functions should be added to generate the set of basis functions at the n-th step. The process is repeated with the enlarged set of functions, and the procedure continues until a small enough error signal is reached.

In [1], the hierarchical generation of the MR functions is described in detail for an arbitrary shape structure, discretized with a rectangular mesh (the technique is not limited to this case); here, we give only a summary.

The MR basis functions are generated by dividing the unknown surface current into its solenoidal (TE) and non-solenoidal (qTM) components, that can be mapped to scalar quantities that are "isotropic," i.e., whjch have the same degree of regularity in all directions of the space. The proposed MR functions are defined first on these isotropic scalar quantities and then mapped back onto their TE and qTM vector counterparts.

The structure is first divided into the minimum possible number of (rectangular) subdomains and the MR basis functions are constructed inside each of them. These subdomains represent the cells of the mesh at the coarsest level, onto which "connecting functions" are generated, that need not be multiresolution.

The generation procedure can be interpreted as the construction of functions of almost identical shape but dilated or contracted to fit onto (nested) meshes of different cell sizes. The obtained set is hierarchical and makes the MR basis functions locally ordered in spatial resolution. This is of great importance in the IMC method, since one knows that adding functions of (one or more) higher levels the accuracy is certainly increased.

III. IMC With Vector MR Functions

In using the MR functions into the IMC scheme, the first issue to be considered is the presence in the solution of the qTM and TE parts, which have completely different behaviors and roles. The iterative adding of MR functions belonging to different levels has to be performed separately for the qTM and TE bases. In the analysis of printed structures, qTM terms are dominant, so one can start to select first the qTM functions and then add the TE, which are responsible for the current behavior near the edges. As concerns the "initial" set, the natural choice is to use the connecting, coarsest-level functions, and, in particular, the qTM ones, which are dominant and defined on the whole structure; they give a first approximation of the solution and account for a large portion of the solution energy.

IMC aims at finding the "minimum" set of necessary basis functions; the associated MoM matrix has a "minimum" order, and the issue of matrix sparsity is not originally considered. The original version of IMC is very efficient for scattering problems, with electrically large bodies with large smooth sections, e.g., a rectangular plate, where fine-level details in the solution appear only in a small portion of the structure, e.g., at the edges of a plate. In typical printed structures, subwavelength-detailed features are very dense, and these details dominate over electrical size even in large structures like printed arrays. This makes the original version of IMC considerably less efficient than in scattering problems, since a lot of fine-scale functions are necessary throughout the structure. However, advantage may be taken here of the fact that basis functions with different scales, as in the MR system, interact via different wave phenomena. Functions belonging to coarser level meshes have spectra localized at low spatial frequencies and, therefore, are the responsible for the coupling between "far" portions of the overall structure (e.g., two radiators in an array). On the contrary, the functions defined on finer meshes are necessary in representing the local and rapid variation of the current (for instance near edges, discontinuities), but they give a small contribution to the coupling between separate elements. This means that the matrix obtained with IMC can be further sparsified by "clipping" the matrix entries that do not give significant contributions. This, however, requires stability with respect to the perturbations introduced by the clipping; this is precisely what the MR scheme in [1] allows. In the following, the "compression ratio" C of the matrix is defined as $C = 1 - N_C^2/N^2$, where N is the order of the "uncompressed" matrix (standard rooftop approach), N_C is the number of iteratively selected functions, i.e., the order of the "compressed" matrix, and N_C^2 is, therefore, the number of entries of this latter. If the compressed matrix is further sparsified, N_C^2 is substituted for the number N_{nz} of nonzero entries of the sparsified compressed matrix (still of dimension $N_C \times N_C$).

As to the numerical efficiency, the described procedure reduces the matrix filling and solution time and the matrix memory occupation. Since the IMC reduces the number of unknowns of a factor N_C/N, the memory occupation of the compressed matrix is $(N_C/N)^2$ times less than with the standard MoM approach. The subsequent sparsification of the matrix further decreases the memory occupation, and both sparsification and compression reduce the linear system solution time. The sparsification can also lead to a further reduction of the fill time, but this requires further manipulation: work on this subject is in progress [5], and not considered here.

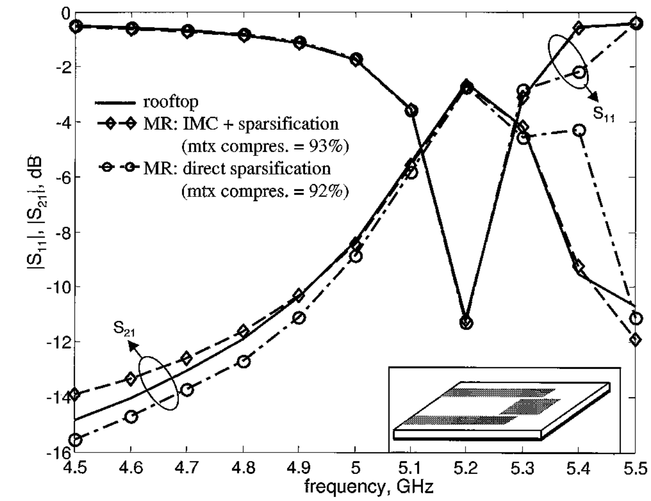

Fig. 1. Bandpass filter. Frequency behavior of S-parameters; solid line: rooftop basis; square marks: MR functions + IMC + sparsification; circle marks: MR functions + sparsification. Inset: analyzed configuration.

Finally, we note that one is usually interested in the frequency behavior of a printed circuit, and, typically, the mesh is the same for all frequency points. We can conjecture that the IMC set of necessary functions is essentially the same for the whole frequency sweep. Thus, the iterative procedure can be applied for solving the problem in the MR basis only at one frequency point (e.g., at the upper end of the band): the reduced number of selected dominant functions are then used for the solution of the problem at all the other frequency points. The consequent saving in the selection overhead results in an increase of the efficiency of the approach. Likewise, the nonzero entries of the sparsified matrix are almost the same over the frequency sweep; therefore, it is sufficient to seek the indices of the matrix entries above the threshold in correspondence to one or two frequency points and then compute only these entries at all the other frequency points.

IV. Results

The MR-IMC scheme has been applied to the analysis of different planar printed structures (on a rectangular mesh). In all cases, the solution obtained with this procedure is compared with that obtained using the conventional subdomain rooftop functions without any matrix compression or sparsification that is taken as reference.

As a first example, we have analyzed the edge-coupled band-pass filter sketched in the inset of Fig. 1. The total number of unknowns used to discretize the current on the filter is rather large, $N = 846$, despite of the simplicity of the structure, but it is necessary to accurately represent the coupling between the three lines that constitute the filter. In fact, the gap between two adjacent lines is $G = 0.024\lambda_g$, where λ_g is the guided wavelength at the frequency $f = 5.2$ GHz (the substrate is 1.6

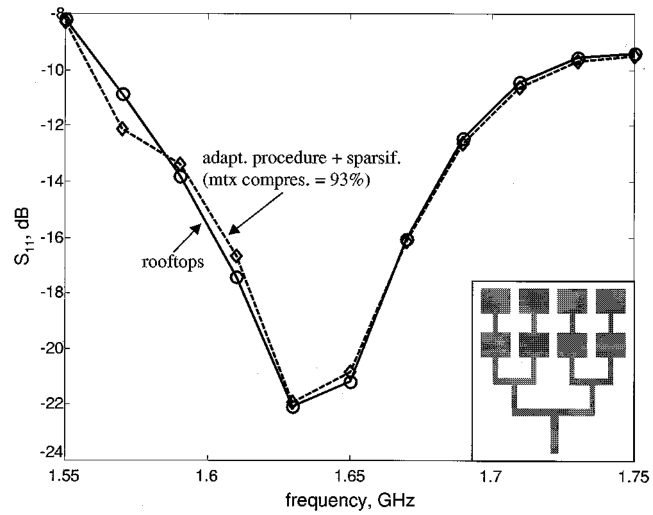

Fig. 2. 4×2 patch array. Frequency behavior of S_{11}; solid line: rooftop basis; the dashed line: MR functions + IMC + sparsification. Inset: analyzed configuration.

mm high and has $\epsilon_r = 2.17$) and the overlap between the access lines and the central conductor is $O = 0.36\lambda_g = 0.75L_c$, where L_c is the total length of the central conductor. Since, in this case, no connecting functions are necessary, the initial set of the functions for the iterative procedure is made by the qTM functions defined on the coarsest rectangular meshes. The IMC procedure reduces the number of unknown of about a quarter, i.e., $N_C \simeq 0.75N$. The limited performance of IMC is essentially due to the reasons explained in Section III, i.e., the compression procedure eliminates the functions that are "redundant" for representing the solution; due to the tight coupling between the lines, a large number of details are necessary to represent the solution and, therefore, all the (high frequency) functions in proximity of the coupling regions have large amplitudes and cannot be discarded. Nevertheless, as pointed out in Section III, the IMC matrix can be further sparsified, setting to zero the entries that represent the interaction between high frequency functions defined on far domains. This sparsification of the compressed matrix strongly reduces the number of nonzero entries (compression ratio $C \simeq 93\%$ in this example), leaving the accuracy of the solution almost unaffected. The scattering parameters obtained after this further matrix manipulation are reported in Fig. 1, together with the reference (rooftop functions). The frequency behavior of $|S_{11}|$ resulting from the application of IMC + sparsification (using the MR basis) is undistinguishable from the reference solution, and the displacement of $|S_{21}|$ is below 0.1 dB in-band and within 0.8 dB at the extreme points of sweep range. For comparison, Fig. 1 also shows the results obtained with the direct sparsification of the uncompressed matrix in MR basis; these refer to the choice of a threshold for matrix sparsification that yields (approximately) the same number on nonzero entries as with the compression + sparsification procedure. The results show that the IMC + sparsification approach is more accurate that the sparsification alone.

Finally, in Fig. 2, the results relative to the 4×2 patch array in the inset are shown. In this case, the number of unknowns in the rooftop basis is $N = 1947$; the IMC procedure reduces them to $N_C = 1300$ and the further sparsification of the compressed matrix leaves only the 7% of nonzero entries, i.e., $C = 93\%$. The curves in Fig. 2 show that the resonance frequency of the structure obtained with this reduced matrix is virtually undistinguishable from the reference, and the values of $|S_{11}|$ are within 0.6 dB from the reference solution.

In the examples shown, IMC yields $(N_C/N)^2 \sim 0.5$, thereby reducing the fill time to $1/2$; the subsequent matrix sparsification reduces the matrix storage to typically $(N_C/N)^2 \sim 10\%$. In addition, the method reduces the computational effort needed to solve the system for each frequency point of almost one order of magnitude of flops (floating point operation) with respect to the reference solution. The overhead needed for the matrix compression that is, however, carried out only once has to then be added; for ten frequency points, the total saving of flops with respect to the solving of the problem in rooftop basis is 50–70%. Note that these are relatively small-size problems, and that the advantage increases with the problem size and the number of involved frequency points.

V. Conclusion

The MR scheme introduced in [1] for printed structures fits well into the iterative impedance matrix compression (IMC) introduced in [2], [3]. The compression procedure is completely automatic, i.e., once the desired level of accuracy is fixed, no user intervention is needed. However, in typical printed structures, the IMC compression rate is moderate. Efficiency is regained by subsequent sparsification of the obtained matrix, which is granted by the properties of the employed MR basis. The results obtained with the compression + sparsification procedure are more accurate than those reachable with direct sparsification of the MoM matrix in the MR basis [1].

Moreover, due to the multilevel structure of the MR functions, in the iterative procedure, the requested accuracy can be modified during the process, without having to restart the process. Note, finally, that the automatic adaptation of the scheme makes it particularly profitable for the analysis of complex structures, with many details, because it automatically adds fine-level functions only in proximity of the most critical regions.

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